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Absorption by extremal D3-branes*

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Abstract: The absorption in the extremal D3-brane background is studied for a class of massless fields whose linear perturbations leave the ten-dimensional background metric unperturbed, as well as the minimally-coupled massive scalar. We find that various fields have the same absorption probability as that of the dilaton-axion system, which is given exactly via the Mathieu equation. We analyze the features of the absorption cross-sections in terms of effective Schrödinger potentials, conjecture a general form of the dual effective potentials, and provide explicit numerical results for the whole energy range. As expected, all partial-wave absorption probabilities tend to zero (one) at low (large) energies, and exhibit an oscillatory pattern as a function of energy. The equivalence of absorption probabilities for various modes has implications for the correlation functions on the field, including subleading contributions on the field-theory side. In particular, certain half-integer and integer spin fields have identical absorption probabilities, thus providing evidence that the corresponding operator pairs on the field theory side belong to the same supermultiplets.

Keywords: p-branes, D-branes, Brane Dynamics in Gauge Theories, AdS/CFT Correspondance.

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Contents

1. Introduction

Scattering processes in the curved backgrounds of p-brane configurations of M-theory and string theory have been extensively studied over the past few years. Motivation for these studies is the fact that the low energy absorption cross-sections for different fields yield information about the two-point correlation functions in strongly coupled gauge theories via AdS/CFT correspondence [1, 2, 3].

The extremal D3-brane background is of special interest, since the correspondence there is to $D = 4$ super Yang-Mills theory. By now, there have been extensive studies of the scattering processes at low-energies for the minimally-coupled scalar field (dilaton-axion) from the gravity perspective, as well as the analyses of the correlation functions on the field-theory side (see, for example, [4]–[10] and references therein), thus leading to important insights into the AdS/CFT correspondence. In particular, the supersymmetry constraints imply that the two-point correlation functions do not get renormalized by higher-order corrections on the field theory side, and a precise agreement between low-energy absorption cross-sections for all the partial waves of the dilaton field and the (weak coupling) field-theory calculations of the corresponding correlation functions was obtained [8].

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On the other hand, the scattering of other fields has been explored to a lesser extent, and that only for low-energies (see, for example, [12, 13] and references therein); nevertheless it is expected that non-renormalization theorems on the field theory side ensure a precise agreement between the low energy absorption cross-sections and the corresponding weak coupling calculation of the n-point correlation functions on the field-theory side (see, for example, [9, 10, 11] and references therein).

This paper addresses several issues. We provide an analysis of the absorption cross-section in the extremal D3-brane background for a broad class of massless modes and for the whole energy range. In particular, we uncover a pattern in the energy dependence of the absorption cross-sections for both integer and half-integer spins; certain half-integer and integer spin pairs have identical absorption cross-sections, thus providing an evidence on the gravity side that such pairs couple on the dual field theory side to the pairs of operators forming supermultiplets of strongly coupled gauge theory.

The paper is organized as follows. In section 2, we cast the wave equations of various fields into Schrödinger form, and obtain the effective Schrödinger potentials. We show that effective Schrödinger potentials for certain field are identical and hence the absorption for these fields is the same. In other cases where Schrödinger potentials are not the same, we argue that the different potentials are dual and yield the same absorption probabilities, with numerical results supporting these claims. In section 3, we obtain numerical results for the absorption of a large class of fields for the whole energy range in the extremal D3-brane background. The method for the numerical evaluation of the absorption probabilities is given in appendix A, while the calculation of the high energy absorption cross-section in the geometrical optics limit is given in appendix B.

2. Effective potentials

The D3-brane of the type-IIB supergravity is given by

$$\begin{aligned} ds_{10}^2 &= H^{1/2}\Big(- f\,dt^2 + dx_1^2 + dx_2^2 + dx_3^2\Big) + H^{1/2}\left(f^{-1}dr^2 + r^2\, d\Omega_5^2\right), \\ G_{(5)} &= d^4x \wedge dH^{-1} + *\left(d^4x \wedge dH^1\right), \end{aligned} \tag{2.1}$$

where

$$H = 1 + \frac{R^4}{r^4}\,, \qquad f = 1 - \frac{2m}{r^4}\,. \tag{2.2}$$

Here R specifies the D3-brane charge and m is the non-extremality parameter (defined for convenience as $m \equiv \mu R^4/\sqrt{1+\mu}$). We shall primarily concentrate on the extreme limit $m = \mu = 0$. (See, however, appendix B for the discussion of the high energy limit of the absorption cross-section in the non-extreme D3-background.)

The low energy absorption probabilities for various bosonic linearly-excited massless fields under this background were obtained in [12]. The low energy absorption probabilities for the gravitino and the two-form field are given in [13] and [14], respectively (and for massive minimally coupled modes in [19]). In the following subsections we study the absorption probabilities for the whole energy range and uncover completely parallel structures. In particular, we shall cast the wave equation for different modes into Schrödinger form and discuss the pattern of the the Schrödinger potentials. We also provide a conjectured form of the dual potentials which, in turn, yield the same absorption probabilities. In the subsequent section we confirm the pattern with numerical results.

2.1 Dilaton-axion

The axion and dilaton of the type-IIB theory are decoupled from the D3-brane. Thus, in the D3-brane background, they satisfy the minimally-coupled scalar wave equation

$$\frac{1}{\sqrt{g}}\partial_\mu \sqrt{g} g^{\mu\nu} \partial_\nu \phi = 0 \,. \tag{2.3}$$

It follows from (2.1) that the radial wave equation of a dilaton-axion in the spacetime of an extremal D3-brane is given by

$$\left(\frac{1}{\rho^5} \frac{\partial}{\partial \rho} \rho^5 \frac{\partial}{\partial \rho} + H - \frac{\ell(\ell+4)}{\rho^2} \right) \phi(\rho) = 0 \,, \tag{2.4}$$

where

$$H = 1 + \frac{e^4}{\rho^4} \tag{2.5}$$

and $\ell = 0, 1, \ldots$ corresponds to the ℓ-th partial wave.

The quantity e and ρ are dimensionless energy and radial distance parameters: $e = \omega R$ and $\rho = \omega r$. The leading order and sub-leading order cross-sections of the minimally-coupled scalar by the D3-brane background were obtained in [4, 6] by matching inner and outer solutions of the wave equations. It was observed in [7] that if one performs the following change of variables

$$\rho = e\, \exp(-z) \,, \qquad \phi(r) = \exp(2z)\, \psi(r) \,, \tag{2.6}$$

the wave equation (2.4) becomes

$$\left[\frac{\partial^2}{\partial z^2} + 2e\, \cosh(2z) - (\ell+2)^2 \right] \psi(z) = 0 \,, \tag{2.7}$$

which is precisely the modified Mathieu equation. One can then obtain analytically the absorption probability order by order in terms of dimensionless energy e [7].

In this paper, we shall express the wave equation in Schrödinger form, and study the characteristics of the Schrödinger effective potential. By the substitution

$$\phi = \rho^{-5/2}\psi\,, \tag{2.8}$$

we render (2.4) into Schrödinger form

$$\left(\frac{\partial^2}{\partial\rho^2} - V_{\rm eff}\right)\psi = 0\,, \tag{2.9}$$

where

$$V_{\rm eff}(\ell) = -H + \frac{(\ell+3/2)(\ell+5/2)}{\rho^2} \equiv V_{\rm dilaton}(\ell)\,. \tag{2.10}$$

Factors shared by the incident and outgoing parts of the wave function cancel out when calculating the absorption probability. Thus, the absorption probability of ϕ and ψ are the same.

Technically, $V_{\rm eff}$ cannot be interpreted as an effective potential, since it depends on the particle's incoming energy. It is straightforward to use a coordinate transformation to put the equation in the standard Schrödinger form, where $V_{\rm eff}$ is independent on the energy. However, for our purposes of analyzing and comparing the form of the wave equations for various fields, this is of no consequence.

Note that the first term in (2.10) represents the spacetime geometry of the extremal D3-brane, whereas the second term represents the angular dynamics (partial modes) of the particles. We shall see presently that some particles have effective potentials which contain terms mixed with both "geometrical" and "angular" dynamical contributions.

2.2 Antisymmetric tensor from 4-form

For two free indices of the 4-form along S^5 and two free indices in the remaining 5 directions, the radial wave equation for the antisymmetric tensor derived from the 4-form is [12]

$$\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} + H - \frac{(\ell+2)^2}{\rho^2}\right)\phi(\rho) = 0\,, \tag{2.11}$$

where $\ell = 1, 2, \ldots$. By the substitution

$$\phi = \rho^{-1/2}\psi\,, \tag{2.12}$$

we render (2.11) into Schrödinger form with

$$V_{\text{4-form}}(\ell) = V_{\rm dilaton}(\ell)\,. \tag{2.13}$$

2.3 Dilatino

The radial wave equation for the dilatino on an extremal D3-brane was found [13] by inserting the following spherical wave decomposition form for the dilatino field λ into the covariant Dirac equation:

$$H^{1/8}\lambda = e^{-i\omega t} r^{-5/2}\left(F(r)\Psi^{\pm}_{-\ell} + iG(r)\left(\frac{\Gamma^0\Gamma^i x_i}{r}\right)\Psi^{\pm}_{-\ell}\right), \tag{2.14}$$

where Γ^i are field-independent gamma-matrices and $i = 4, \ldots, 9$ runs normal to the brane. $\Psi^{\pm}_{-\ell}$ is the eigenspinor of the total angular momentum with $\Sigma_{ij}L_{ij} = -\ell$, $\Gamma^{0123} = \pm i$.

The spatial momenta tangential to the branes can be made to vanish via the Lorentz transformations and the radial wave equations are obtained [13]:

$$\begin{aligned}
\omega H^{1/2} F + \left(\frac{d}{dr} + \frac{\ell + 5/2}{r} \pm \frac{1}{4}(\ln H)'\right) G &= 0\,, \\
-\omega H^{1/2} G + \left(\frac{d}{dr} - \frac{\ell + 5/2}{r} \mp \frac{1}{4}(\ln H)'\right) F &= 0\,.
\end{aligned} \tag{2.15}$$

Decoupling (2.15) yields second-order differential equations for F and G.

Let us first consider the case of positive eigenvalue, i.e. $\Gamma^{0123} = +i$. In this case, the second-order wave equation for F can be cast into Schrödinger form by the substitution $F = H^{1/4}\,\psi$, giving rise to the effective Schrödinger potential

$$V^F_{+\text{dilatino}}(\ell) = V_{\text{dilaton}}(\ell)\,. \tag{2.16}$$

Thus, the absorption probability for F is identically the same as that for the dilaton-axion. The wave equation for G can also be cast into Schrödinger form by the substitution $G = H^{1/4}\,\psi$. The corresponding Schrödinger potential, on the other hand, takes a different form, given by

$$V^G_{+\text{dilatino}}(\ell) = V_{\text{dilaton}}(\ell) + \frac{-3e^8 - 2e^8\ell - 10e^4\rho^4 + 5\rho^8 + 2\rho^8\ell}{H^2\rho^{10}}\,. \tag{2.17}$$

Thus, we see that the effective potentials for the two components F and G are quite different. However, we expect that these two potentials, although different, yield the same absorption probability: they form a dual pair of potentials. While it is not clear to us how to present a rigorous analytical proof, numerical calculations similar to what is presented in section 3 confirm that, indeed, they yield the same absorption probability.

Similar results are obtained for the the negative eigenvalue solutions, i.e. $\Gamma^{0123} = -i$:

$$\begin{aligned}
V^G_{-\text{dilatino}}(\ell) &= V_{\text{dilaton}}(\ell+1)\,, \\
V^F_{-\text{dilatino}}(\ell) &= V_{\text{dilaton}}(\ell+1) + \frac{-5\rho^8 - 2\rho^8\ell - 10e^4\rho^4 + 7e^8 + 2e^8\ell}{H^2\rho^{10}}\,.
\end{aligned} \tag{2.18}$$

Again, numerical calculations similar to what is presented in section 3 indicate that the above two potentials yield the same absorption probabilities and hence form a dual pair.

In the above discussion of the dilatino scattering equation, we encountered three different potentials, namely

$$V^G_{+\text{dilatino}}(\ell) \xrightarrow{\text{dual}} V^F_{+\text{dilatino}}(\ell) = V^G_{-\text{dilatino}}(\ell-1) \xrightarrow{\text{dual}} V^F_{-\text{dilatino}}(\ell-1) \tag{2.19}$$

which all yield the same absorption probability.

The above structure of the dual potentials can be cast in a more general form. Namely, the above dual potential pairs for the dilatino can be cast into the following form:

$$V(\ell) = -H + \frac{(2\ell+\alpha)(2\ell+\alpha\pm 2)}{4\rho^2}\,, \tag{2.20}$$

and

$$V^{\text{dual}}(\ell) = V(\ell) + \frac{\pm[(2\ell+\alpha\pm 2)e^8 - (2\ell+\alpha)\rho^8] - 10e^4\rho^4}{\rho^{10}H^2}\,, \tag{2.21}$$

where $\alpha = 5$ and we use in the $\pm$ sign in (2.21) for the $\mp$ eigenvalue, respectively. We have found numerically that (2.20) and (2.21) are dual potentials for integer values of α. In particular, for odd values of α, (2.20) can be identified with $V_{\text{dilaton}}(\ell+(\alpha-4\pm 1)/2)$, in which case the absorption can be found analytically, since the wave equation is that of Mathieu equation.

Thus, in all the subsequent examples when the potential is of the form (2.21), we are now able to identify a dual potential (2.20) of a simpler form with a corresponding wave equation which can be solved analytically.

2.4 Scalar from the two-form

For the free indices of the two-form taken to lie along the S^5, the radial wave equation is [12]

$$\left(\frac{H}{\rho}\frac{\partial}{\partial\rho}\frac{\rho}{H}\frac{\partial}{\partial\rho} + H - \frac{(\ell+2)^2}{\rho^2} \mp \frac{4e^4}{H\rho^6}(\ell+2)\right)\phi(\rho) = 0\,, \tag{2.22}$$

where again $\ell = 1, 2, \ldots$ correspond to the ℓ-th partial wave. The sign $\pm$ corresponds to the sign in the spherical harmonics involved in the partial wave expansion.

By the substitution

$$\phi = \rho^{-1/2}H^{1/2}\psi\,, \tag{2.23}$$

we render (2.22) into Schrödinger form.

For the positive eigenvalue, the effective potential is of the form given by (2.21), with $\alpha = 5$ and a positive sign. This is, in fact, the same as that for the dilatino with negative eigenvalue. Thus, the dual potential is

$$V_{+\text{scalar}}(\ell) \xrightarrow{\text{dual}} V_{\text{dilaton}}(\ell+1)\,. \tag{2.24}$$

For the negative eigenvalue, the effective potential is of the form given by (2.21), with $\alpha = 3$ and a negative sign. Thus, the dual potential is

$$V_{-\text{scalar}}(\ell) \xrightarrow{\text{dual}} V_{\text{dilaton}}(\ell - 1)\,. \tag{2.25}$$

2.5 Two-form from the antisymmetric tensor

The equations for two-form perturbations polarized along the D3-brane are coupled. For s-wave perturbations, they can be decoupled [12, 14], and the radial wave equation is

$$\left(\frac{1}{\rho^5 H}\frac{\partial}{\partial\rho}\rho^5 H\frac{\partial}{\partial\rho} + H - \frac{16e^8}{\rho^{10}H^2}\right)\phi(\rho) = 0\,, \tag{2.26}$$

where $\ell = 0, 1, \ldots$. By the substitution

$$\phi = \rho^{-5/2}H^{-1/2}\psi\,, \tag{2.27}$$

we render (2.26) into Schrödinger form given by (2.21), with $\alpha = 5$, $\ell = 0$ and the positive sign. Thus, the dual potential is

$$V_{2\text{-form}}(\ell = 0) \xrightarrow{\text{dual}} V_{\text{dilaton}}(\ell = 1)\,. \tag{2.28}$$

2.6 Vector from the two-form

We now consider one free index of the two-form along S^5 and one free index in the remaining 5 directions. For the tangential components of this vector field, the radial wave equation is [12]

$$\left(\frac{1}{\rho^3}\frac{\partial}{\partial\rho}\rho^3\frac{\partial}{\partial\rho} + H - \frac{(\ell+1)(\ell+3)}{\rho^2}\right)\phi(\rho) = 0\,, \tag{2.29}$$

where $\ell = 1, 2, \ldots$. By the substitution

$$\phi = \rho^{-3/2}\psi\,, \tag{2.30}$$

we render (2.29) into Schrödinger form with

$$V_{\text{tangential-vector}}(\ell) = V_{\text{dilaton}}(\ell)\,. \tag{2.31}$$

The radial a_r and time-like a_0 components of the vector field are determined by the following coupled first order differential equations [12]:

$$i\frac{\partial}{\partial\rho}a_o = \left[1 - \frac{(\ell+1)(\ell+3)}{\rho^2 H}\right]a_r \tag{2.32}$$

and

$$i\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{\rho}{H}a_r\right) = a_o\,, \tag{2.33}$$

where $\ell = 1, 2, \ldots$. Equations (2.32) and (2.33) can be decoupled and the wave equation for a_r is

$$\left(H \frac{\partial}{\partial\rho} \frac{1}{\rho} \frac{\partial}{\partial\rho} \frac{\rho}{H} + H - \frac{(\ell+1)(\ell+3)}{\rho^2} \right) a_r = 0 \,. \tag{2.34}$$

By the substitution

$$a_r = \rho^{-1/2} H \psi \,, \tag{2.35}$$

we render (2.34) into Schrödinger form with

$$V_{\text{radial-vector}}(\ell) = V_{\text{dilaton}}(\ell) \,. \tag{2.36}$$

The wave equation for a_0 is

$$\left(\frac{1}{\rho} \frac{\partial}{\partial\rho} \frac{\rho^3}{\rho^2 H - (\ell+1)(\ell+3)} \frac{\partial}{\partial\rho} + 1 \right) a_0 = 0 \,. \tag{2.37}$$

By the substitution

$$a_0 = \rho^{-3/2} \sqrt{\rho^2 H - (\ell+1)(\ell+3)}\, \psi \,, \tag{2.38}$$

we render (2.37) into Schrödinger form with

$$V_{\text{0-vector}}(\ell) = V_{\text{dilaton}}(\ell) + \frac{3e^8 - 10e^4\rho^4 + 4\rho^6(\ell+1)(\ell+3) - \rho^8}{\rho^{10}(H - (\ell+1)(\ell+3)/\rho^2)^2} \,. \tag{2.39}$$

Note that this effective potential is non singular only for $e^2 > (\ell+1)(\ell+3)/2$. We have numerically confirmed that $V_{\text{0-vector}}(\ell)$ is dual to $V_{\text{dilaton}}(\ell)$. Working directly from eqs. (2.32) and (2.33), we have numerically confirmed that a_0 shares the same absorption probability with the dilaton-axion, for all values of e.

3. Qualitative features of absorption by the extremal D3-brane

We have obtained numerical absorption probabilities by a method described in appendix A. Figure 1 shows the s-wave absorption probabilities for all the particles that we have studied on an extremal D3-brane vs. the energy of incoming particles measured in dimensionless units of $e \equiv \omega R$. (a) is the s-wave absorption probability of the dilaton-axion, dilatino with positive total angular momentum eigenvalue and scalar from the two-form with a negative sign in the spherical harmonic. (b) is the s-wave absorption probability of the dilatino with negative total angular momentum eigenvalue, two-form from the antisymmetric tensor, antisymmetric tensor from the

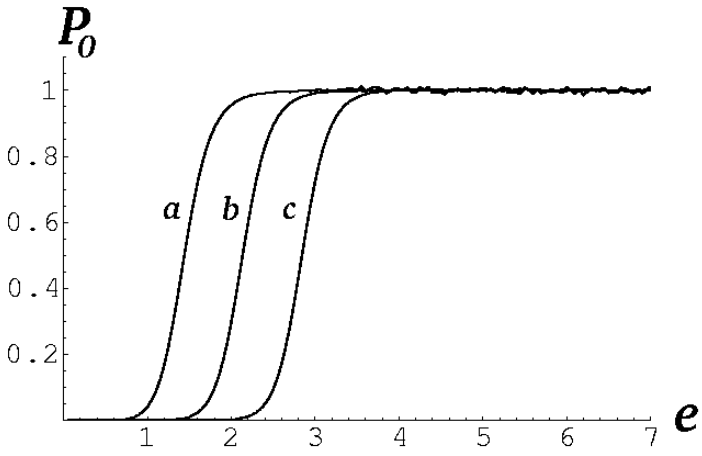

Figure 1: s-wave absorption probabilities of various particles on an extremal D3-brane.

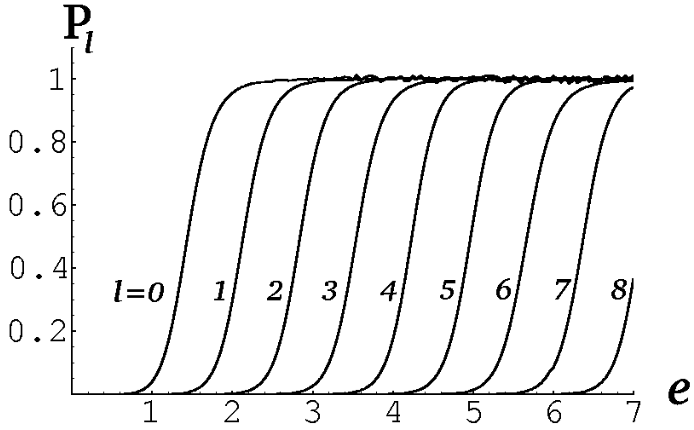

Figure 2: Partial absorption probabilities for a dilaton-axion on an extremal D3-brane.

four-form and the longitudinal and tangential components of the vector from the two-form. (c) is the s-wave absorption probability of the scalar from the two-form with a positive sign in the spherical harmonic. These absorption probabilities are simply related by $\ell \to \ell \pm 1$. This structure is suggestive of a particle supermultiplet structure– namely, different multiplets.

This demonstrates the similar and surprisingly simple structure of absorption probabilities between the various particles, which is not apparent from previous analytical low-energy absorption probabilities. These numerical results also support the idea of dual potentials.

Figure 2 shows the partial absorption probabilities for a dilaton-axion on an extremal D3-brane vs. the energy of incoming particles. There is no absorption at zero energy and total absorption is approached at high energy. Thus, the high energy

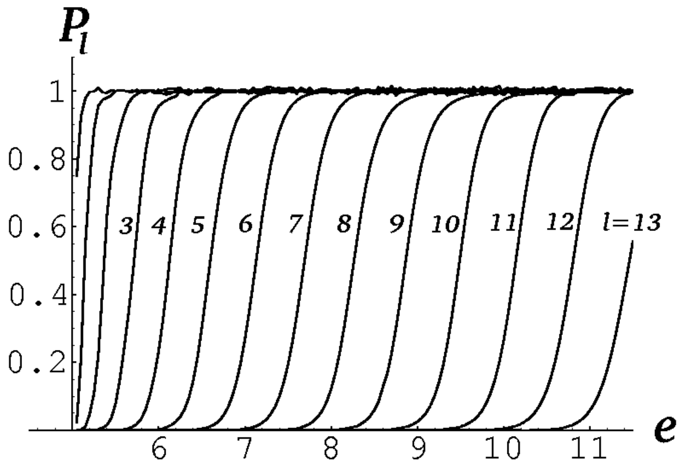

Figure 3: Partial absorption probabilities for a massive minimally-coupled scalar on an extremal D3-brane.

absorption cross-section is found by setting $P = 1$ in (A.4):

$$\sigma^{(\ell)} = \frac{8\pi^2(\ell+3)(\ell+2)^2(\ell+1)}{3(\omega R)^2}\,. \tag{3.1}$$

In fact, for all types of waves absorbed by all branes and black holes studied thus far [16], total partial wave absorption occurs at high energy. It is conjectured that this is a general property of absorption for all objects that have an event horizon. Results for objects other than D3-branes will be published shortly by the present authors.

The partial absorption probabilities of a massive minimally-coupled scalar on an extremal D3-brane vs. the energy of the incoming particles is shown in figure 3. As the mass of the scalar is increased, the partial absorption probabilities occur at lower energies. Physically, there is greater absorption at low energy due to the additional gravitational attraction that is present from the non-zero mass. The low energy absorption probability for this case were obtained in [19].

Figure 4 shows the numerical s-wave absorption cross-section for a dilaton-axion on an extremal D3-brane (continuous line), super-imposed with previously obtained low energy semi-analytical results (short dashes) [7], as well as high energy total absorption (long dashes) vs. the energy of incoming particles. Throughout the rest of this paper, absorption cross-sections are plotted in units of R^5. The resonance roughly corresponds to the region in which there is a transition from zero absorption to total absorption.

As evident from figure 5, the energies at which there is a resonance in the partial absorption cross-section are proportional to the partial-wave number. Also, the magnitude of the peak of each partial absorption cross-section decreases with the

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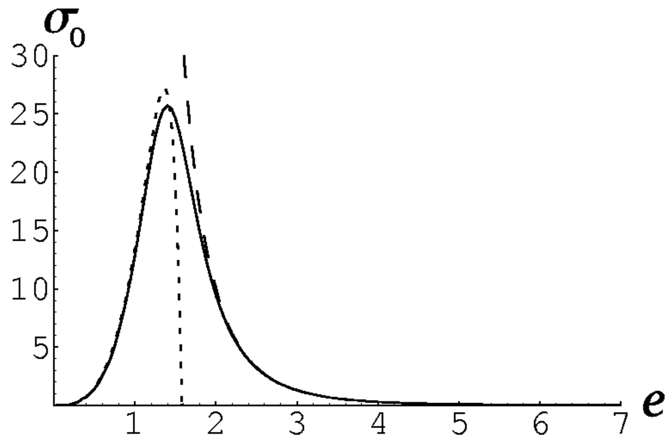

Figure 4: s-wave absorption cross-section for a dilaton-axion on an extremal D3-brane.

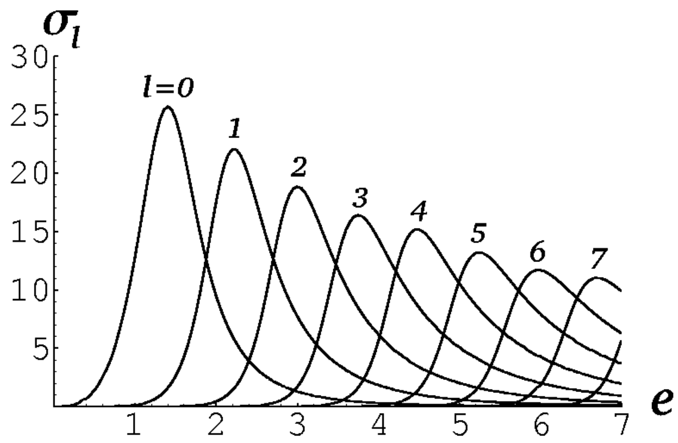

Figure 5: Partial absorption cross-sections for a dilaton-axion on an extremal D3-brane.

partial-wave number. These characteristics seem reasonable if one considers particle dynamics; in terms of radial motion, rotational kinetic energy counteracts gravitational attraction.

In figure 6, we plot the partial-wave effective potentials vs. radial distance (in dimensionless units) for a dilaton-axion on an extremal D3-brane. These are plotted at the energies of the maxima of the corresponding partial-wave absorption cross-sections. For the s-wave absorption cross-section, the maximum is at $e = 1.4$. The maxima of higher partial-wave absorption cross-sections are separated by energy gaps of approximately $\Delta e = .75$, with increasing ℓ.

As can be seen, the incoming particle must penetrate an effective barrier in order to be absorbed by the D3-brane. Once absorbed, the waves inhabit quasi-bound states until they quantum tunnel to asymptotically flat spacetime. As is

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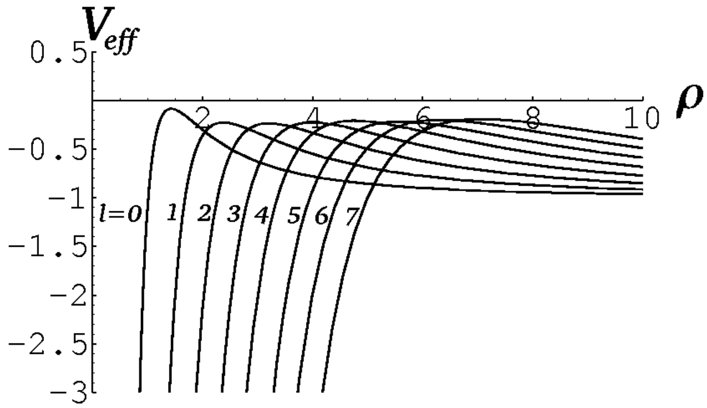

Figure 6: Partial effective potentials absorption for a dilaton-axion on an extremal D3-brane.

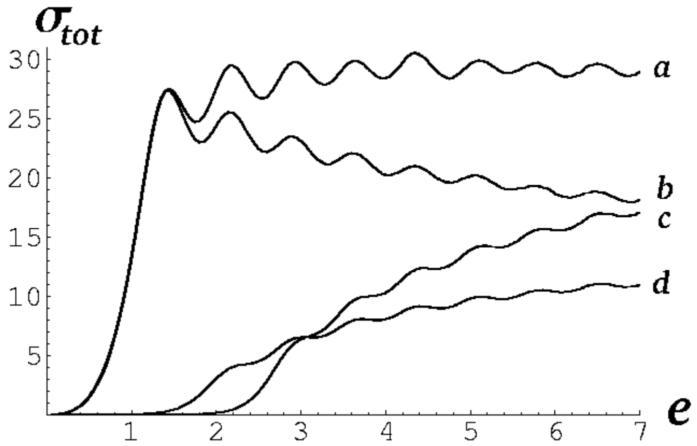

Figure 7: Total absorption cross-sections for the dilaton-axion, dilatino and scalar from two-form on an extremal D3-brane.

shown, the heights of the partial-wave effective potential barriers at the energies of the maxima of the partial-wave absorption cross-sections are roughly equal, which partly explains the similar structure of the partial absorption probabilities of different partial-wave numbers. Thus, the decreasing maximum values of the partial-wave absorption cross-sections with increasing ℓ arises purely as a result of the phase-factors.

The superposition of maxima of partial-wave absorption cross-section leads to the oscillatory character of the total absorption cross-section with respect to the energy of the incoming particles. This is shown in figure 7 for the case of the dilaton-axion, dilatino and scalar from the two-form for comparison. (a) is the total absorption cross-section of the dilaton-axion on an extremal D3-brane, (b) is that of the dilatino with positive total angular momentum eigenvalue, (c) is that of the scalar from the

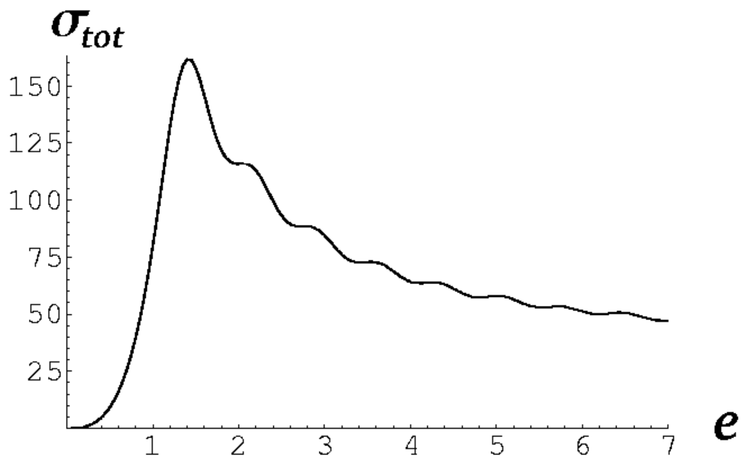

Figure 8: Total absorption cross-section for the scalar from two-form with negative total angular momentum eigenvalue.

two-form with positive sign in the spherical harmonic, and (d) is that of the dilatino with negative total angular momentum eigenvalue.

The amplitude of oscillation decreases exponentially with energy. For the dilaton-axion, the total absorption cross-section converges to the geometrical optics limit at high energy, which we have calculated in a previous section. The oscillatory behavior is shared by all total absorption cross-sections that have been studied as of this time [16], for various particles in various spacetime backgrounds. The oscillatory structure of the absorption cross-section of a scalar on a Schwarzschild black hole has been noted by Sanchez [17].

It is interesting to note that extinction cross-sections which arise in the field of optics have similar oscillatory properties [18].

Figure 8 shows the total absorption cross-section of the scalar from the two-form with negative total angular momentum eigenvalue.

As already noted and shown in figure 3, as the mass of the minimally-coupled scalar is increased, the partial-wave absorption probabilities occur at lower energies. This causes the drastic qualitative difference between the massless and massive scalar cases at low energy, i.e. the divergent total absorption cross-section at zero energy for the massive scalar, as is shown in figure 9.

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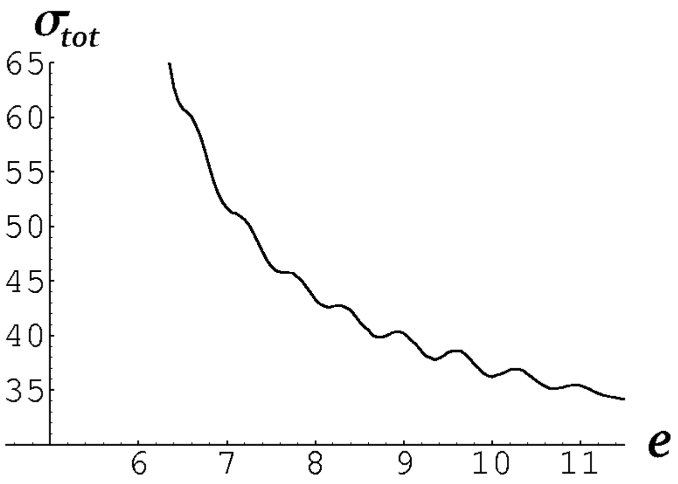

Figure 9: Total absorption cross-section for a massive minimally-coupled scalar on an extremal D3-brane.

A. Outline of numerical method

We will now outline the numerical method for finding the absorption cross-section. We will use the well-studied case of the dilaton-axion on an extremal D3-brane. In this case, the radial wave equation is given by (2.9) and (2.10). We take the wave close to the horizon, at $\rho = .01$, to be purely incoming:

$$\psi(\rho) = \rho \exp\left(\frac{i(\omega R)^2}{\rho}\right) . \tag{A.1}$$

The solution in the far region is:

$$\psi(\rho) = A_{\text{in}} \exp(i\rho) + A_{\text{out}} \exp(-i\rho) . \tag{A.2}$$

We use Mathematica to numerically integrate (2.9) with the boundary condition given by (A.1). At $\rho = 45$, we match the result with (A.2) to find A_{in} and A_{out}. The absorption probability is given by

$$P = 1 - \left|\frac{A_{\text{out}}}{A_{\text{in}}}\right|^2 . \tag{A.3}$$

The absorption cross-section for a scalar is

$$\sigma^{(\ell)} = \frac{8\pi^2(\ell+3)(\ell+2)^2(\ell+1)}{3(\omega R)^2} P^{(\ell)} . \tag{A.4}$$

Numerical integration for other types of particles incident on other objects with event-horizons is straight-forward.

B. High-energy absorption cross-section for a dilaton-axion on a non-extremal D3-brane

In this appendix, we obtain the analytical high energy absorption cross-section for a dilaton-axion in a *non-extremal* D3-brane background, by employing the geometrical optics limit for the classical motion of a particle [15].

For a non-extremal D3-brane, the metric takes the form (2.1). The classical lagrangian for a particle is of the form:

$$L = \frac{1}{2} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta \,. \tag{B.1}$$

$\dot{x}^\alpha \equiv dx^\alpha/d\lambda$, where λ is an affine parameter. The Euler-Lagrange equations are

$$\frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{x}^\alpha}\right) - \frac{\partial L}{\partial x^\alpha} = 0 \,. \tag{B.2}$$

The equation of motion for θ is

$$\frac{d}{d\lambda}\left(H^{1/2} r^2 \dot{\theta}\right) = H^{1/2} r^2 \sin\theta \cos\theta \left(\dot{\phi_3}^2 - \dot{\phi}^2 - \cos^2\phi \dot{\phi_1}^2 - \sin^2\phi \dot{\phi_2}^2\right) . \tag{B.3}$$

The solution of (B.3) is $\theta = \pi/2$ and $\dot{\theta} = 0$. The equations of motion for ϕ_3 and t are of the form

$$\frac{d}{d\lambda}\left(H^{1/2} r^2 \dot{\phi_3}\right) = 0 \tag{B.4}$$

and

$$\frac{d}{d\lambda}(H^{-1/2} f \dot{t}) = 0 \,. \tag{B.5}$$

(B.4) and (B.5) each imply a constant of motion:

$$H^{1/2} r^2 \dot{\phi_3} = \text{constant} \equiv \ell \tag{B.6}$$

and

$$H^{-1/2} f \dot{t} = \text{constant} \equiv E \,, \tag{B.7}$$

where ℓ and E are interpreted as the angular momentum and energy of the particle, respectively.

Also, since the particle scatters only in a direction transverse to the D3-brane (i.e. it does not travel along the D3-brane), $\dot{x_i} = 0$, for $i = 1, 2, 3$. Substituting our solutions for θ and $\dot{x_i}$ into the lagrangian yields

$$2L = -H^{-1/2} f \dot{t}^2 + H^{1/2} f^{-1} \dot{r}^2 + H^{1/2} r^2 \dot{\phi_3}^2 \,. \tag{B.8}$$

Instead of finding a rather complicated equation of motion for r, we use the fact that, for a massless particle,

$$2L = g_{\alpha\beta} p^\alpha p^\beta = 0 \,, \tag{B.9}$$

where $p^\alpha = \dot{x}^\alpha$. Substituting our results for $\dot{\phi}_3$ and $\dot{t}$ into the previous equation yields

$$\dot{r}^2 = E^2 - \frac{\ell^2}{r^2}\frac{f}{H}\,. \tag{B.10}$$

We introduce a new parameter, $\lambda' \equiv \ell\lambda$, so that

$$\left(\frac{dr}{d\lambda'}\right)^2 = \frac{1}{b^2} - V_{\text{effective}}\,, \tag{B.11}$$

where $b \equiv \ell/E$ is the impact parameter and

$$V_{\text{effective}} = \frac{1}{r^2}\frac{f}{H}\,. \tag{B.12}$$

The absorption cross-section for particles at high energy can be obtained by determining the classical trajectory of the scattered particle and using the optical limit result:

$$\sigma_{\text{abs}} = \frac{8}{15}\pi^2 b_{\text{crit}}^5\,, \tag{B.13}$$

where the critical impact parameter separating absorption from scattering orbits is given by $1/b_{\text{crit}}^2 = V_{\text{maximum}}$. Thus,

$$\begin{aligned} \sigma_{\text{abs}} &= \frac{8}{15}\pi^2\left(\frac{1}{2}R^4 + 3m + \frac{1}{2}\sqrt{(R^4+6m)^2 + 8mR^4}\right)^{5/4} \\ &= \left(\frac{\frac{3}{2}R^4 + 3m + \frac{1}{2}\sqrt{(R^4+6m)^2+8mR^4}}{\frac{1}{2}R^4 + m + \frac{1}{2}\sqrt{(R^4+6m)^2+8mR^4}}\right)^{5/2}\,. \end{aligned} \tag{B.14}$$

In the extremal limit, i.e. $m = 0$, this result reduces to:

$$\sigma_{\text{abs}} = \frac{32\sqrt{2}}{15}\pi^2 R^5\,. \tag{B.15}$$

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